Math 53: Multivariable Calculus **Sections 212, 213**

Worksheet for 2020-09-23

Problem 1. Here are some conceptual questions on the second derivative test. Let D be the Hessian determinant. When performing the second derivative test, there are a number of cases you consider (see last page of Lecture 08 slides):

- $D(a,b) > 0$ and $f_{xx}(a,b) > 0$
- $D(a,b) > 0$ and $f_{xx}(a,b) < 0$
- $D(a,b) < 0$

Regarding this procedure:

(a) What's the conclusion in each scenario?

(b) In the first two scenarios, why do we look at f_{xx} instead of f_{yy}? (Is there a difference?)

(c) The case $D(a,b) = 0$ is missing. Explain why it's missing.

(d) The case $D(a,b) > 0$ and $f_{xx}(a,b) = 0$ is also missing. Explain why it's missing.

(e) Can you find an example of a function $f(x,y)$ such that $(0,0)$ is a critical point of f, $f_{xx}(0,0)$ and $f_{yy}(0,0)$ are both strictly positive, but $(0,0)$ is *not* a local minimum?

Problem 2 (Stewart §14.7 #22)**.** Consider the function $f(x, y) = x^2 y e^{-x^2-y^2}$. Show that

(a) Show that $(\pm 1, 1/\sqrt{2})$ are local maxima and $(\pm 1, -1/\sqrt{2})$ are local minima.

(b) **Actually f has infinitely many other critical points. Find them, and classify them.